T6A044

"Reliability – Staggered Proof Testing Coefficients"

1 Contents

2 Revision History

Version	Date	Author	Comments
0.1	28/01/2022	R. Martin	Draft release for public comment.
0.2	25/01/2023	R. Martin	Minor corrections to text.
0.3	16/08/2023	R. Martin	Minor corrections to text. Foreword and Executive Summary updated.
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3 Introduction / Foreword

This document covers the topic of staggered proof testing in relation to functional safety and is part of a series of documents linking together to support a reliability calculation tool.

There are four documents in the series:

1. Reliability and Availability
2. Effects of Proof Testing
3. Fault Tolerant Systems
4. Staggered Proof Testing Coefficients

This document builds on the concepts developed in *Reliability and Availability* and *Effects of Proof Testing*.
Reliability and Availability explores the basic mathematics of reliability and explains:

- What is meant by constant failure rate;
- The effect of parallel and series networks;
- The relationship between λ and MTBF;
- The importance of repairable systems and Availability (as an average over time);
- The time average likelihood of being in a failed state (so called PFDAV);
- The other terms in common use for detected and undetected failures;
- The differing effects of detected and undetected failures;
- The effects of common proof testing regimes on multiple failures;
- The effects of common cause failures
- Simple and complex redundancy;
- Conditional Probability;
- Estimating reliability from data.

Effects of Proof Testing explores:

- The basic effects of synchronous proof testing.
- The basic effects of staggered proof testing.

Fault Tolerant Systems pulls the developed ideas from the earlier documents to create an algorithm for calculating the system PFD and failure rate for an MooN fault tolerant system taking into account:

- detected, undetected and residual failures ('residual' covers the effect of an incomplete proof test);
- common cause factors for detected, undetected and residual failures;
- synchronous and staggered proof testing.

The resulting formulae (for MooN system failure rate and PFD) have been constructed as spreadsheet formulae covering 1oo1, 1oo2, 1oo3, 2oo3 and 2oo4 systems. These standard forms are on the workbook sheet labelled 'Calc Sheet'. For other MooN configurations, the workbook has a sheet labelled 'N-f': this uses vba to calculate results.

This document is to support *Effects of Proof Testing* in the topic area of Staggered Proof Testing.

This considers the failure of a subset with the understanding that the average probability of failure of the subset depends on their relative positions in the testing cycle. The approach taken was to use an analytical method to generate an algorithm for finding the PFD for the general case of a subset selection; to consider all possible subsets; and then average the results.

The analytical evaluation of a subset is carried out by one program. The determination of all possibilities and the addition and the division are carried out by another program making calls on the first. This resulted in the generation of a matrix of coefficients for staggered testing which are captured on a separate sheet within the PFD Calculator workbook. It is a one-off use: the only reason to use it again is to generate higher order coefficients. However, the code that generates the staggered proof test coefficients is included in the PFD Calculator workbook for posterity.

This document represents the authors notes on the algorithm development used in creating the VBA routines to generate the coefficients required by *Staggered Proof Testing*. It is nothing more than an aide memoire.

The final result of these coefficients is used in *Fault Tolerant Systems* which is the document which lies behind the *PFD Calculator* workbook.

4 Executive Summary

The development of this series of documents came as a result of *The 61508 Association* (T6A) setting up a working group (WG) to produce good practice guidance on 'SIL Assessment' (the assessment of the ability of a system to perform a required safety function with the required integrity).

The history of the development is as follows:

- T6A set up WG15 to produce a good practice guide for 'SIL Assessment'.
- It became apparent that a spreadsheet would be the most suitable tool to use because of its ability with computational calculations and the ease of access and familiarity to most people.
- It also became apparent that the spreadsheet needed a 'built in' reliability calculator so that all important reasoning could be separated from number crunching but also that 'verification' in any instance of use would be confined to the reasoning and the appropriate use of the calculator rather than the calculator itself. So, it was decided to create the 'built in' calculator.
- Before creating the calculator, it became necessary to produce the formulae upon which the calculator would be based.
- Reliability is taught at many higher educational establishments and there is much information on safety related systems calculations in circulation. However, the authors were unable to find a source that pulled it all together into general formulae. A document entitled 'Fault Tolerant Systems' was therefore created covering the development of the necessary formulae for calculating the failure rate and the probability of failure for so called 'MooN' fault tolerant systems.
- The formulae developed catered for diagnosed and undiagnosed failures, distortion due to synchronous proof testing and common cause failures.
- However, when the document was being verified, it became clear that verifiers needed some further explanation of the maths and (importantly) the development of the necessary terminology.
- Over time, it emerged that limited proof test coverage was becoming an issue of interest (especially to regulators). It also emerged that staggered proof testing for higher order systems gave considerable 'on paper' benefits. So, it was decided to add these two features to the calculator.
- As a result, three further documents were considered necessary:
    - One that covered the theory from first principles (now entitled Reliability and Availability).
    - One that covered the distorting effects of synchronous and staggered proof testing on the calculations (now entitled Effects of Proof Testing).
    - Because finding the distorting effects of staggered testing proved to be quite complex (a mixture of analytical and numerical techniques were used) it was decided to make the deduction of the staggered testing coefficients

into a separate documents (now entitled Staggered Proof Testing Coefficients).

The formulae have now been developed from first principles and the spreadsheet calculator produced. The documents and the calculator have been independently verified.

5 Terminology

f	General term for 'fault tolerance' – i.e. for simple redundancy, the number of failed devices a system can tolerate and still perform its function. Note: r is the general term for the number of survivors required for a system to perform its function.
F	Probability of failure (normally a function of time). Note: this has the same meaning at PFD (probability of failure on demand).
MT	Mission Time (for use with residual failures)
MTBF	Mean time before failure. $\mathrm{MTBF} = 1/\lambda$ (for constant λ)
MTTR	Mean time to restore.
PFD	Probability of failure on demand. Notes: • this has the same meaning as F (probability of failure). • This is sometimes used in the text as shorthand for PFD_{AV}.
PFD_{AV}	Time average of PFD.
PFD_D	PFD for diagnosed failures for single channel / device. $PFD_D = PFD_D^1 = \left((1-\beta_D)\lambda dd.MTTR\right)$
PFD_R	PFD for residual failures for single channel / device. $PFD_R = PFD_R^1 = \left(\frac{(1-\beta_R)\lambda drMT}{2}\right)$
PFD_U	PFD for undiagnosed failures for single channel / device. $PFD_U = PFD_U^1 = \left(\frac{(1-\beta_U)\lambda duT}{2}\right)$
PFD_D^k	PFD for diagnosed failures for k channels / devices $PFD_D^k = (PFD_D)^k$
PFD_R^k	PFD for residual failures for k channels / devices $PFD_R^k \neq (PFD_R)^k$ due to test regime
PFD_U^k	PFD for undiagnosed failures for k channels / devices

	$PFD_U^k \neq (PFD_U)^k$ due to replacement regime
PFD^N	PFD rolled up for all failures for N channels / devices (including common causes)
R	Probability of survival (normally a function of time).
s	Used as a suffix to represent attributes of a system. E.g. F_S is used to represent probability of system failure.
T	Proof test interval.
β	Beta factor – general term for fraction of failures which affect all channels / devices.
β_D	Beta factor specific to diagnosed failures
β_D	Beta factor specific to residual failures
β_U	Beta factor specific to undiagnosed failures
λ	General term for underlying failure rate – a function of time that represents the failure rate 'given that there is no current failure'. This document assumes it is a constant in time. Note: this is not the same as $\dot{F}(t)$ (which is the failure rate not assuming current survival).
λ_d	General term for diagnosed failure rate – i.e. failure that is automatically revealed.
λ_u	General term for undiagnosed failure rate.
λ_{dd}	Dangerous diagnosed failure rate.
λ_{dr}	Dangerous residual failure rate – i.e. dangerous failure rate that is not automatically revealed or revealed by periodic proof test.
λ_{du}	Dangerous undiagnosed failure rate.

6 Complex redundancy with staggered proof testing

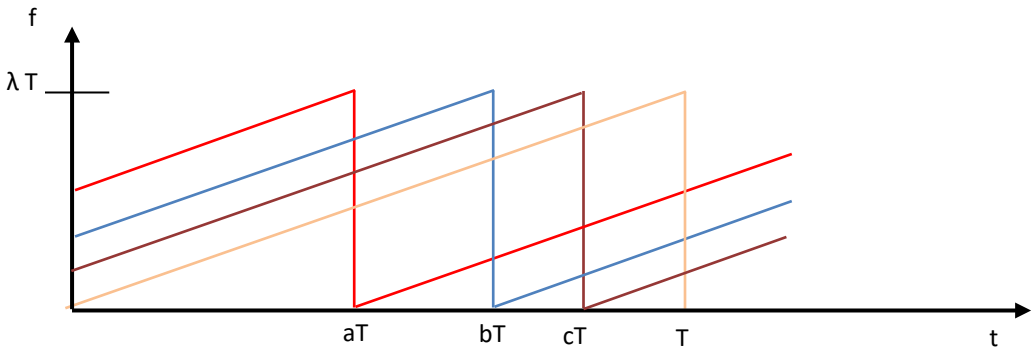
Here, we consider the case of complex redundancy where faults are undiagnosed (i.e. found only in proof testing) and where the system fails if M out of N items fail.

The purpose of this document is to generate the algorithms that will generate a 10 by 10 table of numbers representing the test factors to be applied. The test factors are multiplying factors for M out of N failures where these factors are multiplied by $PFD_{1oo1}{}^{N}$ to find the PFD_{AV}.

The mathematics is dealt with in more detail in Staggered Proof Testing but the cases of 4 failures and M failures are repeated here for information.

7 Failure of 4 Items

Consider the following graph



The joint probability of failure is given by

$$F(t) = \lambda t(\lambda t + (1-a)\lambda T)(\lambda t + (1-b)\lambda T)(\lambda t + (1-c)\lambda T)$$
[0,aT]

$$F(t) = \lambda t(\lambda t + (-a)\lambda T)(\lambda t + (1-b)\lambda T)(\lambda t + (1-c)\lambda T)$$
[aT,bT]

$$F(t) = \lambda t(\lambda t + (-a)\lambda T)(\lambda t + (-b)\lambda T)(\lambda t + (1-c)\lambda T)$$
[bT,cT]

$$F(t) = \lambda t(\lambda t + (-a)\lambda T)(\lambda t + (-b)\lambda T)(\lambda t + (-c)\lambda T)$$
[cT,T]

We write this as: $F(t) = x(x+A)(x+B)(x+C)$

Where
$x = \lambda t;$

$A = (1-a)\lambda T\ [t < aT]$
$A = (-a)\lambda T\ [t \geq aT]$
B= $(1-b)\lambda T\ [t < bT]$
B= $(-b)\lambda T\ [t \geq bT]$
$C = (1-c)\lambda T\ [t < cT]$
$C = (-c)\lambda T\ [t \geq cT]$

Expanding the expression for F(t)

$$F(t) = x(x^2 + (A+B)x + AB)(x + C)$$

$$F(t) = x(x^3 + (A+B+C)x^2 + (AB+AC+BC)x + ABC)$$

$$F(t) = x^4 + (A+B+C)x^3 + (AB+AC+BC)x^2 + ABCx$$

Note: The coefficients for powers of x (other than the first) are:

- the sum of all the solos, then
- the sum of all the pairs, then
- the sum of all the triples

This pattern is repeated for greater powers.

So, for 4 failures with staggered proof testing:

$$F(t) = \lambda^4 t^4 + (A+B+C)\lambda^3 t^3 + (AB+AC+BC)\lambda^2 t^2 + ABC\lambda t$$

$$F_{AV} = \frac{1}{T}\left[\lambda^4 \frac{t^5}{5} + (A+B+C)\lambda^3 \frac{t^4}{4} + (AB+AC+BC)\lambda^2 \frac{t^3}{3} + ABC\lambda \frac{t^2}{2}\right]_0^{aT}$$
$$+\frac{1}{T}\left[\lambda^4 \frac{t^5}{5} + (A+B+C)\lambda^3 \frac{t^4}{4} + (AB+AC+BC)\lambda^2 \frac{t^3}{3} + ABC\lambda \frac{t^2}{2}\right]_{aT}^{bT}$$
$$+\frac{1}{T}\left[\lambda^4 \frac{t^5}{5} + (A+B+C)\lambda^3 \frac{t^4}{4} + (AB+AC+BC)\lambda^2 \frac{t^3}{3} + ABC\lambda \frac{t^2}{2}\right]_{bT}^{cT}$$
$$+\frac{1}{T}\left[\lambda^4 \frac{t^5}{5} + (A+B+C)\lambda^3 \frac{t^4}{4} + (AB+AC+BC)\lambda^2 \frac{t^3}{3} + ABC\lambda \frac{t^2}{2}\right]_{cT}^{T}$$

Where
$A = (1-a)\lambda T\ [t < aT]$
$A = (-a)\lambda T\ [t \geq aT]$

B= $(1-b)\lambda T$ $[t < bT]$
B= $(-b)\lambda T$ $[t \geq bT]$

$C = (1-c)\lambda T$ $[t < cT]$
$C = (-c)\lambda T$ $[t \geq cT]$

We could simplify this whole expression by replacing as follows:

$A' = A/\lambda T$

$B' = B/\lambda T$

$C' = C/\lambda T$

Then, for 4 failures with staggered proof testing:

$$F_{AV} = \frac{\lambda^4}{T}\left[\frac{t^5}{5} + (A'+B'+C')\frac{Tt^4}{4} + (A'B'+A'C+B'C')\frac{T^2t^3}{3} + A'B'C'\frac{T^3t^2}{2}\right]_0^{aT}$$
$$+\frac{\lambda^4}{T}\left[\frac{t^5}{5} + (A'+B'+C')\frac{Tt^4}{4} + (A'B'+A'C+B'C')\frac{T^2t^3}{3} + A'B'C'\frac{T^3t^2}{2}\right]_{aT}^{bT}$$
$$+\frac{\lambda^4}{T}\left[\frac{t^5}{5} + (A'+B'+C')\frac{Tt^4}{4} + (A'B'+A'C+B'C')\frac{T^2t^3}{3} + A'B'C'\frac{T^3t^2}{2}\right]_{bT}^{cT}$$
$$+\frac{\lambda^4}{T}\left[\frac{t^5}{5} + (A'+B'+C')\frac{Tt^4}{4} + (A'B'+A'C+B'C')\frac{T^2t^3}{3} + A'B'C'\frac{T^3t^2}{2}\right]_{cT}^{T}$$

Where:
$A' = (1-a)$ $[x < a]$
$A' = (-a)$ $[x \geq a]$
$B' = (1-b)$ $[x < b]$
$B' = (-b)$ $[x \geq b]$
$C' = (1-c)$ $[x < c]$
$C' = (-c)$ $[x \geq c]$

Changing variables and limits to simplify:

$$F_{AV} = \lambda^4 T^4\left[\frac{x^5}{5} + (A'+B'+C')\frac{x^4}{4} + (A'B'+A'C+B'C')\frac{x^3}{3} + A'B'C'\frac{x^2}{2}\right]_0^{a}$$
$$+\lambda^4 T^4\left[\frac{x^5}{5} + (A'+B'+C')\frac{x^4}{4} + (A'B'+A'C+B'C')\frac{x^3}{3} + A'B'C'\frac{x^2}{2}\right]_a^{b}$$

$$+\lambda^4 T^4 \left[\frac{x^5}{5} + (A' + B' + C')\frac{x^4}{4} + (A'B' + A'C + B'C')\frac{x^3}{3} + A'B'C'\frac{x^2}{2}\right]_b^c$$

$$+\lambda^4 T^4 \left[\frac{x^5}{5} + (A' + B' + C')\frac{x^4}{4} + (A'B' + A'C + B'C')\frac{x^3}{3} + A'B'C'\frac{x^2}{2}\right]_c^1$$

Where:

$A' = (1 - a)\ [x < a]$
$A' = (-a)\ [x \geq a]$
$B' = (1 - b)\ [x < b]$
$B' = (-b)\ [x \geq b]$
$C' = (1 - c)\ [x < c]$
$C' = (-c)\ [x \geq c]$

8 Failure of N Items

It is possible to expand and find the general case from the above.

$$F_{AV} = \left(\frac{\lambda T}{2}\right)^N 2^N \left[\frac{x^{N+1}}{N+1} + (Sum\ of\ solos)\frac{x^N}{N} + (Sum\ of\ pairs)\frac{x^{N-1}}{N-1} + (Sum\ of\ triples)\ \frac{x^{N-2}}{N-2} + \ldots.\right]_0^a$$

$$+ \left(\frac{\lambda T}{2}\right)^N 2^N \left[\frac{x^{N+1}}{N+1} + (Sum\ of\ solos)\frac{x^N}{N} + (Sum\ of\ pairs)\frac{x^{N-1}}{N-1} + (Sum\ of\ triples)\ \frac{x^{N-2}}{N-2} + \ldots.\right]_a^b$$

$$+ \cdots$$

$$+ \left(\frac{\lambda T}{2}\right)^N 2^N \left[\frac{x^{N+1}}{N+1} + (Sum\ of\ solos)\frac{x^N}{N} + (Sum\ of\ pairs)\frac{x^{N-1}}{N-1} + (Sum\ of\ triples)\ \frac{x^{N-2}}{N-2} + \ldots.\right]_{..}^1$$

Note: $\left(\frac{\lambda T}{2}\right)$ is F_{AV} for 1oo1

Where:

$A' = (1 - a)\ [x < a]$
$A' = (-a)\ [x \geq a]$
$B' = (1 - b)\ [x < b]$
$B' = (-b)\ [x \geq b]$
$C' = (1 - c)\ [x < c]$

$C' = (-c)\ [x \geq c]$
and:

$$a < b < c < d \ldots \ldots < 1$$

Finding the products of singles, pairs, triples etc in a set

To be able to evaluate the above, we need to be able to find the sum of the products of pairs, triples, quads etc.

8.1.1 Set of 3

Assume we have a vector of 3 elements: x_1, x_2 and x_3
The sum of the singles is: $x_1 + x_2 + x_3$
The sum of pairs is: $x_1x_2 + x_1x_3 + x_2x_3$
The sum of triples is: $x_1x_2x_3$

8.1.2 Set of 4

The sum of the singles is: $x_1 + x_2 + x_3 + x_4$

The sum of pair products is: $x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4$

The sum of triple products is: $x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4$

The sum of quad products is: $x_1x_2x_3x_4$

The sum of the singles is: $x_1 + x_2 + x_3 + x_4$

The sum of pairs is: $x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4$

This is x_1 times (sum of singles in range x_2 to x_4) + x_2 times (sum of singles in range x_3 to x_4) + x_3 times (sum of singles in range x_4 to x_4)

The sum of triples is: $x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4$

This is x_1 times (sum of pairs in range x_2 to x_4) + x_2 times (sum of pairs in range x_3 to x_4)

The sum of quads is: $x_1x_2x_3x_4$

This is x_1 times (sum of triples in range x_2 to x_4)

8.1.3 Set of 5

The sum of the singles is: $x_1 + x_2 + x_3 + x_4 + x_5$

The sum of pair products is:

$$x_1x_2 + x_1x_3 + x_1x_4 + x_1x_5 + x_2x_3 + x_2x_4 + x_2x_5 + x_3x_4 + x_3x_5 + x_4x_5$$

This is x_1 times (sum of singles in range x2 to x5) + x_2 times (sum of singles in range x_3 to x_5) + x_3 times (sum of singles in range x_4 to x_5) + x_4 times (sum of singles in range x_5 to x_5)

The sum of triple products is:

$$x_1x_2x_3 + x_1x_2x_4 + x_1x_2x_5 + x_1x_3x_4 + x_1x_3x_5 + x_1x_4x_5 + x_2x_3x_4 + x_2x_3x_5 + x_2x_4x_5 + x_3x_4x_5$$

This is x_1 times (sum of pairs in range x_2 to x_5) + x_2 times (sum of pairs in range x_3 to x_5) + x_3 times (sum of pairs in range x_4 to x_5)

The sum of quad products is:

$$x_1x_2x_3x_4 + x_1x_2x_3x_5 + x_1x_2x_4x_5 + x_1x_3x_4x_5 + x_2x_3x_4x_5$$

This is x_1 times (sum of triples in range x_2 to x_5) + x_2 times (sum of triples in range x_3 to x_5)

The sum of the quint products is: $x_1x_2x_3x_4x_5$

This is x_1 times (sum of quads in range x_2 to x_5)

Note, the above in blue text can all be found algorithmically.

The sum of the singles is: $x_1 + x_2 + x_3 + x_4 + x_5$

The sum of pair products is:

$$x_1(x_2 + x_3 + x_4 + x_5)$$
$$+x_2(x_3 + x_4 + x_5)$$
$$+x_3(x_4 + x_5)$$
$$+x_4(x_5)$$

The sum of triple products is:

$$x_1x_2(x_3 + x_4 + x_5)$$
$$+x_1x_3(x_4 + x_5)$$
$$+x_1x_4(x_5)$$
$$+x_2x_3(x_4 + x_5)$$
$$+x_2x_4(x_5)$$
$$+x_3x_4(x_5)$$

The sum of quad products is: $x_1x_2x_3(x_4 + x_5)$

$+x_1x_2x_3(x_5)$

$+x_1x_2x_4(x_5)$

$+x_1x_3x_4(x_5)$

$+x_2x_3x_4(x_5)$

The sum of quint products is: $x_1x_2x_3x_4(x_5)$

9 Finding sums of products by programming

9.1.1 The sum of pairs (selections of 2)

To find sums of pairs (selections of 2) in range x_K to x_M
For example: x_4 to x_7
K=4, M=7, N=2

The sum is:

$x_4.x_5 + x_4.x_6 + x_4.x_7$
$+ x_5.x_6 + x_5.x_7$
$+ x_6.x_7$

Written another way, the sum is:

$x_4(x_5+x_6+x_7) + x_5(x_6+x_7) + x_6(x_7)$

i.e.:
x_4 times (sum of singles in range x_5 to x_7)

+ x_5 times (sum of singles in range x_6 to x_7)

+ x_6 times (sum of singles in range x_7 to x_7)

i.e.
x_K times (sum of singles in range x_{K+1} to x_M)

+ x_{K+1} times (sum of singles in range x_{K+2} to x_M)

+.....

+ x_{M-1} times (sum of singles in range x_M to x_M)

To generate this in a program:

Sum = 0

For i = K to M-1

For j = K+1 to M

Product = $x_i.x_j$

Sum = Sum + Product

Next j

Next i

9.1.2 **The sum of triples (selections of 3)**

To find sums of triples (selections of 3) in range x_K to x_M
For example: x_5 to x_8
K=5, M=9, N=3

Sum is
x_5 times($x_6.x_7 + x_6.x_8 + x_6.x_9 + x_7.x_8 + x_7.x_9 + x_8.x_9$) + x_6 times($x_7.x_8 + x_7.x_9 + x_8.x_9$) + x_7 times($x_8.x_9$)
i.e.:

x_5 times (sum of pairs in range x_6 to x_9)

+ x_6 times (sum of pairs in range x_7 to x_9)

+ x_7 times (sum of pairs in range x_8 to x_9)

9.1.3 **The sum of selections of N (in range K to M)**

To generate this algorithmically:

```
For i1 = K To M + 1 - N
      product(1) = x(i1)
      If N > 1 Then
         For i2 = i1 + 1 To M + 2 - N
            product(2) = product(1) * x(i2)
            If N > 2 Then
               For i3 = i2 + 1 To M + 3 - N
                  product(3) = product(2) * x(i3)
                  sum = sum + product(3)
               Next i3
```

```
            Else
               sum = sum + product(2)
            End If
         Next i2
      Else
         sum = sum + product(1)
      End If
   Next i1
```

Where the nesting would be continued to the maximum level required.

10 Finding subsets of M from N

In the following, we are finding all the possible subsets of M integers in the range 1 to N. What seems an easy enough task when N = 3 and M = 2, soon becomes very difficult when these values are higher. A system is required.

10.1 **Subsets of 4 in the range 1 to 7**

Below is a systematic progression for 4 integers in the range 1 to 7.

1	2	3	4
1	2	3	5
1	2	3	6
1	2	3	7
1	2	4	5
1	2	4	6
1	2	4	7
1	2	5	6
1	2	5	7
1	2	6	7
1	3	4	5
1	3	4	6
1	3	4	7
1	3	5	6
1	3	5	7
1	3	6	7
1	4	5	6
1	4	5	7
1	4	6	7
1	5	6	7
2	3	4	5
2	3	4	6
2	3	4	7
2	3	5	6
2	3	5	7

2	3	6	7
2	4	5	6
2	4	5	7
2	4	6	7
2	5	6	7
3	4	5	6
3	4	5	7
3	4	6	7
3	5	6	7
4	5	6	7

We started with 1234 and then indexed the 4th column until it reached the limit 7.
We then indexed the 3rd column (3 to 4) and started by resetting the 4th column to one higher.
We repeated the indexing of the 3rd column until it could go no higher, i.e. the 4th column started at 7.
We then indexed the second column (2 to 3) and started by resetting the 3rd and 4th columns in ascending values.
We repeated the above process until indexing the second column meant that it could go no higher because all columns to the right were packed and so on.

10.2 **Subsets of N in the range 1 to M**

It is possible to generalise the above process.
Set the first column to 1 and then reset all columns to the right.
Note that resetting all columns to the right means make them ascend in turn from the current column's contents.
Index the Mth column until the contents is equal to N.
Index the (M-1)th column and 'reset' column to right.
Index from the Mth column until the contents is equal to N.
Index the (M-1)th column again and repeat the above process until the contents is equal to N-1.
Then start indexing the (M-2)th column and repeat.
On the next page is a routine which generates these combinations.

```
For N = 1 To M
   count = 0
   K = N - 1
   For i1 = 1 To M - K
      S(1) = i1
      If N > 1 Then
         For i2 = i1 + 1 To M - K + 1
            S(2) = i2
            If N > 2 Then
               For i3 = i2 + 1 To M - K + 2
                  S(3) = i3
                  If N > 3 Then
                     For i4 = i3 + 1 To M - K + 3
                        S(4) = i4
                        If N > 4 Then
                        Else
                           count = count + 1
                           Write (S)
                        End If
                     Next i4
                  Else
                     count = count + 1
                     Write (S))
                  End If
               Next i3
            Else
               count = count + 1
```

```
            Write (S)
          End If
        Next i2
      Else
        count = count + 1
        Write (S)
      End If
    Next i1
  Next N
```

11 References

1. IEC 61508, *Functional safety of electrical / electronic / programmable electronic (E/E/PE) safety related systems*, Parts 1-7, 2010 (includes EN and BS EN variants).
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3. ISA-TR84.00.02-2002 Part 1, *Safety Instrumented Functions (SIF) – Safety Integrity Level (SIL) Evaluation Techniques – Part 1: Introduction*, 2002.
4. ISA-TR84.00.02-2002 Part 2, *Safety Instrumented Functions (SIF) – Safety Integrity Level (SIL) Evaluation Techniques – Part 2: Determining the SIL of a SIF via Simplified Equations*, 2002.
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6. SINTEF A11612, *Unrestricted Report – Use of the PDS Method for Railway Applications*, June 2009.
7. Reliability Maintainability and Risk (10th Edition) – Dr David J Smith.
8. New approach to SIL verification – Mirek Generowicz of I&E Systems Pty – Australia (available free to download from *The 61508 Association* website).

12 Conclusion

This detail and the content from the previous three documents on reliability / availability, effects of proof testing and fault tolerant systems has enabled the creation of a Microsoft Excel based reliability calculation tool.

13 Existing and Emerging Standards

IEC 61508:2010 (Series of standards, Edition 2).
IEC 61511-1:2017+A1:2017 (Edition 2).

14 61508 Association Recommended Practices

This document sets out to describe current best practices in *Reliability and Availability* for functional safety systems, but does not seek to prescribe specific measures, since these will depend on the application, and any existing constraints of the installation.

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